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# Particle Mass Spectrum and Strong Gravity

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*Abstract*

Regarding the strong gravity as a source of hadronic interactions, the masses of several stable particles and resonances, including the newly found $\psi$-resonances, are derived from a simple, semiclassical dimensional analysis. Particles are constructed successively from each others in a bootstrap-like manner. A particle property "dusk" is introduced, which in the cases of stable baryons is identified with strangeness. The theoretical background of the construction procedure is discussed.

## 1. *Introduction*

Although the role of gravitation in elementary particle physics has been studied for many decades, only recently did the suppression of ultraviolet divergence in electromagnetic self-energy of an electron by inclusion of gravitational self-energy give a new importance for gravity studies in the elementary particle scale (Isham et al. 1971b). In addition, the discovery of the massive $f$-meson led Isham et al., (1971a) to propose the idea of strong gravity (SG), where the $f$-meson is interpreted as a carrier of the SG in hadronic interaction (see also Salam, 1974).

Assuming the results of general relativity are applicable in the case of SG, some authors (Lord et al., 1974; Tennakone, 1974) have developed particle theories on this SG basis. They use as their starting point the Reissner-Nordström metric (see Hawking and Ellis, 1973) for a particle of mass $m$ and charge $e$, i.e.,

$$ds^2 = -\gamma^{-1}\, dr^2 - r^2\, d\theta^2 - r^2 \sin^2\theta\, d\phi^2 + \gamma c^2\, dt^2 \qquad (1.1)$$

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where

$$\gamma = 1 - 2Gm/(c^2 r) + Ge^2/(c^4 r^2) \tag{1.2}$$

and $G$ is the gravitational constant.

The "Bangalore group" (Lord et al., 1974) uses the proton as their fundamental black hole, and this gives for the SG coupling constant $G$ the value $G = 6.7 \times 10^{30}$ dyne cm$^2$ g$^{-2}$. Other particles are generated as oscillatory excitations of this fundamental black hole. The group has recently developed their theory (Sivaram and Sinha, 1974) along the lines given by Motz (1972) and have obtained many interesting and promising results.

Also Tennakone (1974) uses the Reissner-Nordström solution as his starting point. He postulates the only stable massive particles, the proton and the electron, to be the black hole singularities of the metric (1.1). Inserting in equation (1.2) $\gamma = 0$ and equating the event horizon (Hawking and Ellis, 1973) $r_+$ (the larger root of the equation $\gamma = 0$) with the Compton wavelength of the pion, the proton mass gives for the SG coupling constant $G = 3.78 \times 10^{31}$ dyne cm$^2$ g$^{-2}$. The second root of $\gamma = 0$ is then $r_-$ $7.7 \times 10^{-17}$ cm, which corresponds to the Schwarzschild black hole (Hawking and Ellis, 1973; Misner et al., 1973) mass $m = 0.51$ MeV/$c^2$, et al., the mass of an electron. However, this interpretation of the electron is suspect, since the two different radii $r_+$ and $r_-$ belong to a single Reissner-Nordström solution and since the surface corresponding to $r_-$ cannot be seen through the horizon $r_+$ from outside.

On the other hand, as a particle model, the theory developed by Tennakone is successful. The muon is constructed simply as a system of an electron and a neutral pion, the latter being concentrated near the electron surface. The $e\pi^0$ system has the mass

$$m_\mu = m_e + m_\pi + V + T \tag{1.3}$$

The Newtonian approximation and the virial theorem for the inverse square force are used to calculate the SG potential energy $V$ and the kinetic energy $T$, i.e.,

$$V = -Gm_e m_\pi / rc^2 \tag{1.4}$$

and

$$T = -\tfrac{1}{2}V \tag{1.5}$$

Thus for the mass of the muon one obtains

$$m_\mu = m_e + m_\pi - \tfrac{1}{2}V \tag{1.6}$$

which gives the correct experimental mass ratio $m_\mu/m_e$ (Tennakone, 1974).

Motivated by the work of Tennakone we give in this paper some results of a simple semiclassical dimensional analysis for the masses of elementary particles. We do not build a detailed model. On the contrary, it has been our aim to be as general as possible. We build particles successively from each others in the manner in which Tennakone built the muon. Each composition represents a mode of a particle. We quantize the SG field by postulating that only discrete values of the binding energy in equation (3.1) are allowed. This is done by introducing the so-called baryonic and mesic radii, which represent various modes of interactions. These radii are used in equation (3.1) as if they were ordinary Newtonian distances, even though this is not their true physical meaning.

Indeed, on many occasions Newtonian formulas hold even in the relativistic case, only the meaning of some quantities may change. Thus a Schwarzschild black hole obeys the Newtonian equation of motion if seen from infinity (Demianski and Grishcuk, 1974). Further, the static relativistic two-body configuration obeys the ordinary Newtonian gravitation force law (Bach and Weyl, 1922). In the nonstatic two-body configurations, the existence of the gravitational radiation can nullify the validity of Newtonian-like formulas [equation (3.1)] (Cooperstock, 1974). However, we assume that the SG bound particle systems do not radiate in a stationary state. This is quite an analogous assumption to Bohr's quantum postulate. Therefore we spectulate that the use of Newtonian-like mass formula (3.1) is justified even in a relativistic case.

## 2. *Scales of Length*

We accept from the model of Tennakone (1974) only the value of the constant $G$, and the idea of using the classical virial theorem. In this paper we use MeV as a unit of mass, so that the SG constant is

$$G = 7.50 \times 10^{-1} \text{ cm MeV}^{-1}. \tag{2.1}$$

We define the baryonic distance $r_b$ so that the energy in $nn$ system bound by the SG corresponds to the mass of the neutral pion, when the distance between constituents is $2r_b$, i.e.,

$$\tfrac{1}{2}Gnn/2r_b = \pi_0 \tag{2.2}$$

where the symbols of particles are particle masses in MeV units.

Equation (2.2) yields for the baryonic distance the value

$$r_b = 1.23 \times 10^{-13} \text{ cm} \tag{2.3}$$

This is roughly the same as the SG Schwarzschild radius of the black hole neutron or the Compton wavelength of the pion.

Next we use the old Sakata model for the definition of the mesic distance $r_m$. As a model of a pion we propose a $N\bar{N}$ pair bound by the SG so that

$$N + \bar{N} - \tfrac{1}{2}GN\bar{N}/2r_m = \pi \tag{2.4}$$

This gives for the mesic distance the value

$$r_m = 0.95 \times 10^{-14} \text{ cm} \tag{2.5}$$

It is interesting to observe that the SG Schwarzschild radius $r_s = 2G\pi$ of the pion is two times the mesic distance.

### 3. *Baryon Meson Systems*

In this and the following chapters we give some numerical results obtained from simple calculations for masses of particle systems, formed by the SG. Our fundamental equation, for the mass $M$ of the system of two particles $m_1$ and $m_2$, is

$$M = m_1 + m_2 \pm \tfrac{1}{2}Gm_1m_2/r \tag{3.1}$$

where $G$ is the SG coupling constant and $r$ is the distance between the constituents (i.e., $r$ is a simple combination of the radii of section 2).

In our calculations we use freely also the positive energy (+ sign corresponds to repulsion!). Because we cannot give any reasonable explanation for this "black hole physics" a new property *dusk* ($d$) is introduced: Every time positive energy is used in the construction of more complex systems, the dusk increases by one unit.

In the following tables the masses for some baryon meson systems with possible interpretations are given. In Table 1 the possible candidates for stable baryons are given. The massive states are really many-body systems, but they are treated in this approximation as two-body configurations. If, for example, one imagines the $\Omega$-hyperon to be a superposition of ($\Xi\pi$) and ($\Lambda K$) systems with the relative weights 0.4 and 0.6, respectively, the weights calculated from the 40 known events, one obtains the experimental mass of the $\Omega$-particle.

Comparing the sequence of $d:s$ in Table 1 with the strangeness quantum numbers ($s$) of the stable baryons we obtain the relationship

$$-d = s \tag{3.2}$$

Table 2 contains results for other baryon meson systems with different interaction distances ($r_b$, $2r_m$). As one observes, at distances ($r_b$, $2r_m$) other baryon–meson systems yield masses in the region of baryonic resonances.

In the previous calculations we have used only two different distances, but

TABLE 1. Baryon–meson systems

| System | rep/attr | $d$ | $r$ | $M$ | Identification |
|---|---|---|---|---|---|
| $N\pi$ | rep | 1 | $r_b$ | 1118 | $\Lambda(1115.6)$ |
| $\Lambda\pi$ | attr | 1 | $r_b$ | 1208 | $\Sigma(1112.5)$ |
| $\Lambda\pi$ | rep | 2 | $r_b$ | 1303 | $\Xi(1315)$ |
| $\Xi\pi$ | rep | 3 | $r_b$ | 1510 | $\Omega(1672)$ |
| $\Lambda K$ | rep | ? | $r_b$ | 1777 | $\Omega(1672)$ |

TABLE 2. Baryon-mesons systems (continued)

| System | rep/attr | $r$ | $M$ | Identification |
|---|---|---|---|---|
| $N\bar{K}$ | rep | $r_b$ | 1573 | $\Lambda(1520)$ |
| $N\bar{K}$ | rep | $2r_m$ | 2346 | $\Lambda(2350)$ |
| $N\rho$ | rep | $r_b$ | 1930 | $\Delta(1940, 1950)$ |
| $N\rho$ | rep | $2r_m$ | 1818 | $\Delta(1890)$ |
| $\Lambda\bar{K}$ | rep | $r_b$ | 1783 | $\Xi(1820)$ |
| $\Lambda\eta$ | rep | $r_b$ | 1851 | $\Lambda(1830)$ |
| $\Lambda\omega$ | rep | $r_b$ | 2164 | $\Lambda(2100)$ |
| $\Sigma\pi$ | rep | $r_b$ | 1380 | $\Sigma(1385)$, $\Lambda(1405)$ |
| $\Sigma\pi$ | rep | $2r_m$ | 1657 | $\Sigma(1670)$, $\Lambda(1690, 1670)$ |
| $\Sigma\bar{K}$ | rep | $r_b$ | 1862 | $\Delta(1890, 1910)$, $\Xi(1820)$ |
| $\Xi\pi$ | rep | $r_b$ | 1517 | $\Xi(1530)$ |
| $\Xi\pi$ | rep | $2r_m$ | 1816 | $\Xi(1820)$ |
| $\Xi K$ | rep | $r_b$ | 2020 | $\Sigma(2030)$ |

a richer spectrum is available, if we use, for example the "kaonic distance" $r_k$ defined by the equation

$$N + \bar{N} - \tfrac{1}{2}GN\bar{N}/r_k = K + \bar{K} \tag{3.3}$$

### 4. *Baryon–Antibaryon Systems*

The baryon-antibaryon systems form an exotic region, most of them being in the mass region of the heavy meson resonant states (1790-9000 MeV). We could identify some very interesting states at baryonic and mesic distances; these configurations are listed in Table 3. The most surprising result is, that

TABLE 3. Baryon–antibaryon sytems

| System | rep/attr | $r$ | $M$ | Identification |
|---|---|---|---|---|
| $N\bar{N}$ | attr | $2r_m$ | 136 | $\pi(136)$ "Sakata" |
| $N\bar{N}$ | rep | $2r_m$ | 3622 | $\psi_2(3684)$ |
| $N\bar{\Lambda}$ | rep | $2rm$ | 4120 | $\psi_3(4150)$ |
| $\Lambda\bar{\Sigma}$ | rep | $2r_m$ | 4924 | $\psi_4(4900)$ ? |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $\Omega\bar{\Omega}$ | rep | $2r_m$ | 8863 | ? |
| $N\bar{N}$ | attr | $2r_b$ | 1742 | $X^-(1795)$ ? |
| $N\bar{N}$ | rep | $2r_b$ | 2010 | $S(1930)$ |
| $N\bar{\Lambda}$ | rep | $2r_b$ | 2213 | $K^*(2200)$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $\Omega\bar{\Omega}$ | attr | $2r_b$ | 2919 | $\psi_1(3095)$ |
| $\Omega\bar{\Omega}$ | rep | $2r_b$ | 3770 | $\psi_2(3684)$ |

the "dusk-dual" of the pion has the mass of the recently found particle $\psi(3684)$. The same property is in the $\Omega\bar{\Omega}$ system, the dusk-duals having, approximately, the masses of the $\psi(3095)$ and the $\psi(3684)$.

## 5. *Meson–Meson Systems*

For the *MM* systems we use the hypothesis that mesons can interact at distances $r_m$ and $2r_m$. The decaying modes of some mesons, e.g., $K$, $\eta$, $\omega$, indicate that one should consider three-pion systems as possible models of these particles. Two different spatial configurations (stable equilateral triangle configuration and a linear, equidistance configuration) with slightly different energies are possible.[1]

It is outside the scope of the present article to consider general many-body configurations systematically. However, we mention that the system of two mesic $N\bar{N}$ pairs has the mass of the $\rho$-meson, if the separation between two pairs is the kaonic distance $r_k$ defined by equation (3.3).

In Table 4 some masses of the *MM* systems are compiled.

TABLE 4. Meson–meson systems

| System | rep/attr | $r$ | $M$ | Identification |
|---|---|---|---|---|
| $\pi\pi\pi$ | rep | $2r_m$ | 500 | $K(495)$ |
| $\pi K$ | rep | $2r_m$ | 769 | $\rho(770)$ |
| $\pi\rho$ | rep | $r_m$ | 1334 | $A_2(1310)$ |
| $\pi\rho$ | rep | $2r_m$ | 1122 | $A_1(1100)$ |
| $\pi\omega$ | attr | $r_m$ | 491 | $K(495)$ |
| $\pi\omega$ | rep | $2r_m$ | 1138 | $A_1(1100)$ |
| $\pi f$ | rep | $r_m$ | 2100 | $\rho(2100)$ ? |
| $\pi\psi_1$ | rep | $r_m$ | 4950 | $\psi_4(4900)$ ? |
| $\pi\psi_1$ | attr | $2r_m$ | 2385 | $U(2360)$ |
| $\pi\psi_1$ | rep | $2r_m$ | 4094 | $\psi_3(4150)$ |
| $\pi\psi_2$ | rep | $2r_m$ | 4859 | $\psi_4(4900)$ ? |
| $K\bar{K}$ | rep | $2r_m$ | 1468 | $X_0(1430), X_1(1460)$ |
| $Kf$ | rep | $2r_m$ | 3002 | $\psi_1(3095)$ |
| $\rho f$ | rep | $2r_m$ | 3970 | $\psi_3(4150)$ |
| $\omega f$ | rep | $2r_m$ | 4015 | $\psi_3(4150)$ |

## 6. *Systems with Leptons*

By postulating that leptonic mass is also a source of SG we can explain the mass difference between a neutron and a proton (see also Tennakone, 1974).

[1] The two different lifetimes of the neutral kaons might be connected with these two different spatial configurations.

TABLE 5. Leptonic systems

| System | rep/attr | $r$ | $M$ | Identification |
|---|---|---|---|---|
| $p^+e^-$ | rep | $2r_m$ | 939.7 | $n$ |
| $ne^+$ | attr | $r_m$ | 938.2 | $p^+$ |

The neutron-positron system has total energy very close to the mass of the proton, and in terms of this naive model it seems to be possible that a positron can stabilize a neutron to form a proton. This is, naturally, against the principle of conservation of lepton number. However, this objection can be avoided if a neutrino is also ejected in the combination process. It is interesting to note that a neutron can be explained as a proton-electron system with a repulsive force. This is in accordance with the decay properties of the neutron.

## 7. *Conclusions*

In our analysis, we have neither attempted to calculate any corrections to masses nor to find the best values of our parameters to minimize the errors. Considering the extreme simplicity of our method, the results obtained are quite satisfactory. The general level of this work is that of a bare dimensional analysis, and the use of the SG theory has been more a working hypothesis.

However, quite generally, if a gravitational charge, i.e., mass-energy, is the source of the strong interactions, the corresponding field equations would be nonlinear bootstrap-like, as the Einstein equations are. The mass of a particle would then depend on masses of other particles. Indeed, the method we have used has been a simple bootstrap: We have constructed different modes of a particle with the help of other particles.

On the other hand, if the mass-energy were not the source of the hadronic interactions, one would expect the mass spectrum to resemble the corresponding electromagnetic charge spectrum.

An interesting result is the connection (3.2) between the dusk property and strangeness. We do not claim that the connection (3.2) holds in all circumstances, and further elaboration is needed to solve this question.

A possible objection to the dusk property might be that in the case of repulsion, corresponding to the energy $-(T + V)$, the kinetic energy is negative also. However, the term $T + V$ has only a heuristic meaning, and we could as well think that this term represents only a static potential with a different scale of length.

Our model resembles Bohr's semiclassical model of the hydrogen atom. However, we are not able to give an explicit quantum rule. Bekenstein (1974) has quantized the Kerr black hole by noticing that its surface area is an adiabatic invariant and thus can be quantized in the same way as the action integral in old quantum mechanics. The same thing could be done in our model if only the corresponding quantities were known in a general relativistic two-body problem (see, however, Cadež, 1974).

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